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LAMINAR FORCED CONVECTION HEAT TRANSFER IN CURVED PIPES WITH UNIFORM WALL TEMPERATURE

MITSUNOBU AKIYAMA and K. C. CHENG
Department of Mechanical Engineering, University of Alberta, Edmonton, Alberta, Canada

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NOMENCLATURE

a,	radius of pipe;
C,	constant, $(a^3/4\nu\mu)(\partial P_0/R_c\partial\Omega)$;
$\bar{h}$,	average heat transfer coefficient;
K,	Dean number, $Re(a/R_c)^{\frac{1}{2}}$;
k,	thermal conductivity;
M, N,	number of divisions in R and ϕ directions;
Nu,	Nusselt number, $\bar{h}(2a)/k$;
P_0,	axial pressure measured along the centerline and a function of $R_c\Omega$ only;
Pr,	Prandtl number, ν/α;
Q,	a parameter, $(K^2Pr)^{\frac{1}{4}}$;
$R, \phi, R_c\Omega$,	cylindrical coordinates;
R_c,	radius of curvature of a curved pipe;
Re,	Reynolds number $(2a)\bar{W}/\nu$;
r,	dimensionless radial coordinate, R/a;
r_c,	dimensionless radius of curvature of a curved pipe, R_c/a;
T,	local temperature;
T_w,	uniform wall temperature;
T_0,	uniform fluid temperature at thermal entrance;
U, V, W,	velocity components in R, ϕ and $R_c\Omega$ directions;
u, v, w,	dimensionless velocity components in r, ϕ and $r_c\Omega$ directions.

Greek letters

α,	thermal diffusivity;
θ,	dimensionless temperature difference;
ν,	kinematic viscosity.

Subscript and superscript

w,	value at wall;
0,	value for straight pipe;
$^-$,	average value.

1. INTRODUCTION

THE EXISTING heat transfer results in the literature for fully developed laminar forced convection in curved pipes with uniform wall temperature are rather limited and incomplete in some respects in comparison with the case of uniform wall heat flux [1, 2]. The problem was approached by Maekawa [3] using a perturbation method applicable only to extremely low Dean number flow regime which is practically not important. On the other hand, Mori and Nakayama's approximate solution [4] based on boundary layer approximation near the wall is valid only for high Dean number regime and Prandtl number of order one. Recently, Dravid *et al* [5] presented a numerical result for thermal entrance region heat transfer in curved pipes with uniform wall temperature for the case of Dean number $K = 225$ and $Pr = 5$ only. The purpose of this note is to present an accurate heat transfer result for uniform wall temperature case with Dean number ranging from small to an order of

100, and also point out some heat transfer characteristics which are not brought out by earlier investigations.

2. THEORETICAL ANALYSIS AND NUMERICAL SOLUTION

The assumptions employed in the formulation of the problem and the governing equations are identical to those given in the earlier work [2]. However, the present problem of fully developed laminar forced convection in curved pipes with uniform wall temperature is formulated as the asymptotic case of the thermal entrance region heat transfer (Graetz problem) and as such the energy equation now becomes parabolic as compared with the elliptic problem for the uniform wall heat flux case [2]. Consequently, the numerical method used in [2] cannot be applied to the present problem.

Introducing the following non-dimensional variables and the characteristic parameters, and referring to Fig. 1 of [2], $R = [a]r$, $R_c = [a]r_c$, $U = [v/a]u$, $V = [v/a]v$, $W = [Cv/a]w$, $(T - T_w) = [T_0 - T_w]\theta$, $(a^3/4v\mu)(\partial P_0/R_c\partial\Omega) = C$, and $v/\alpha = Pr$, the energy equation and the boundary condition can be written as

$$u\frac{\partial\theta}{\partial r} + \frac{v}{r}\frac{\partial\theta}{\partial\phi} + \left(\frac{C^2}{r_c}\right)^{\frac{1}{2}}\left(\frac{1}{r_c}\right)^{\frac{1}{2}} w\frac{\partial\theta}{\partial\Omega} = \frac{1}{Pr}\left(\frac{\partial^2\theta}{\partial r^2} + \frac{1}{r}\frac{\partial\theta}{\partial r} + \frac{1}{r^2}\frac{\partial^2\theta}{\partial\phi^2}\right) \quad (1)$$

$$\theta = 0 \text{ at } r = 1. \quad (2)$$

The limiting case of $Pr \to 0$ is of considerable practical interest. For this purpose, the energy equation [2] can be normalized by introducing additionally $\Omega = \Omega_c\omega$ with Ω_c denoting a characteristic value indicating an axial angle Ω required for the thermal entrance length. By noting that the axial convection term must be of order one in the normalized equation, one obtains

$$Pr\left(u\frac{\partial\theta}{\partial r} + \frac{v}{r}\frac{\partial\theta}{\partial\phi}\right) + w\frac{\partial\theta}{\partial\omega} = \frac{\partial^2\theta}{\partial r^2} + \frac{1}{r}\frac{\partial\theta}{\mathrm{d}r} + \frac{1}{r^2}\frac{\partial^2\theta}{\partial\phi^2} \quad (3)$$

$$\Omega_c = Pr(C/r_c) = Pr\,(a/R_c)^{\frac{1}{2}}(K/2\bar{w}).$$

It is seen that when $Pr \to 0$, the convective terms involving Pr can be neglected and the secondary flow effect appears indirectly through w in the axial convection term. It is significant to note that Ω_c provides a measure of the thermal entrance length. The energy equation also suggests that Nusselt number is a function of Prandtl and Dean numbers, and the curvature ratio a/R_c does not appear explicitly.

The details on the numerical solution of the general Graetz equation (1) or (3) using an alternating direction implicit method and the results of various numerical experiments carried out in assessing the convergence and the accuracy of the numerical results are too extensive to be given in this note. For the present purpose, it suffices to mention that the asymptotic Nusselt number value is ascertained by the relative variation of the Nusselt number along axial direction using the following relationship:

$$\varepsilon = (Nu^{(n+c)} - Nu^{(n)})/Nu^{(n)} < 5 \times 10^{-3} \quad (4)$$

where n is an axial step number and c is a reasonably large integer such as 20 or 50.

The mesh sizes of $M = N = 14$, $M = 14$ and $N = 28$, $M = 28$ and $N = 14$, and $M = N = 56$ are examined to establish the accuracy for the numerical results and the mesh size of $M = N = 28$ is found to be satisfactory from the viewpoint of accuracy and computing time. The angular coordinate Ω is transformed into η by using the equation $\eta = 1 - e^{-\Omega}$ and the axial step sizes $\Delta\Omega = 0{\cdot}1 \times 10^{-4}$–$0{\cdot}2 \times 10^{-1}$ corresponding to $\Delta\eta = 0{\cdot}1 \times 10^{-4}$–$0{\cdot}1 \times 10^{-2}$ are used.

3. HEAT TRANSFER RESULTS

3.1 *Temperature field characteristics*

The temperature profiles for the present uniform wall temperature case for various Dean numbers are characteristically different from those of uniform wall heat flux case [2] and are shown in Fig. 1 for comparison. The change in sign of the curvature for the temperature profile in the central region from a negative value for $K = 0$ (straight tube) to a positive value for K = finite is caused by the fact that the convective terms $u\,\partial\theta/\partial r$ and $v\,\partial\theta/r\partial\phi$ are dominant over the axial convection term $w\,\partial\theta/r_c\partial\Omega$ which is always negative for heating case.

3.2 *Nusselt number*

The asymptotic Nusselt number, $Nu = \bar{h}(2a)/k$, can be obtained by considering either the average wall temperature gradient or the overall energy balance in the axial direction as,

$$(Nu)_1 = 2\bar{w}\,|\overline{\partial\theta/\partial r}|_w/|\overline{w\theta}|$$
$$(Nu)_2 = Pr(C^2/r_c)^{\frac{1}{2}}(1/r_c)^{\frac{1}{2}}\,\bar{w}/\overline{w\partial\theta/\partial\Omega}\,|/|\overline{w\theta}|. \quad (5)$$

The evaluation of the mean values appearing in equation (5) is carried out by using Simpson's rule except for $|\overline{w\partial\theta/\partial\Omega}|$ where trapezoidal rule is used.

The effect of Dean number on the angular distribution of local Nusselt number along the pipe wall is shown in Fig. 2 for $Pr = 0{\cdot}7$. A comparison between Fig. 2 and the corresponding plot shown in Fig. 8 of [2] for the case of uniform wall heat flux reveals that the local Nusselt number ratio for the case of uniform wall temperature is consistently higher than that for the case of uniform wall heat flux with the same Dean number; such as $K = 123{\cdot}2$ shown in both figures, but the general trend is similar.

The overall heat transfer results in terms of Nusselt number ratio vs. Dean number are shown in Fig. 3 for

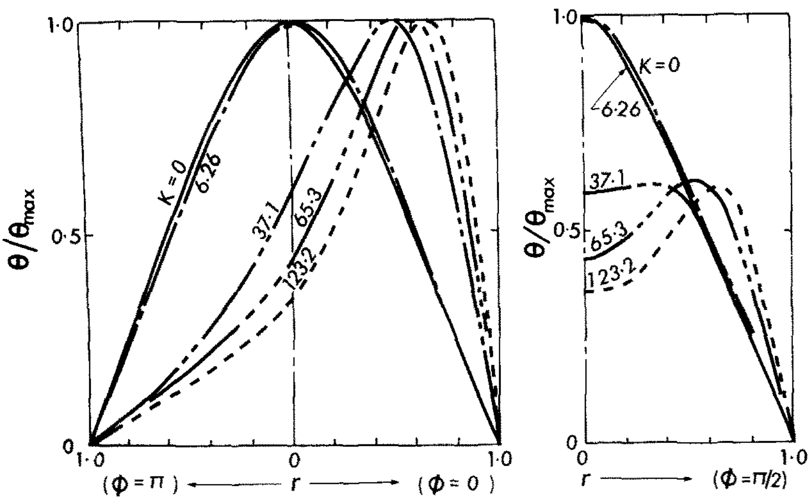

FIG. 1. Temperature distributions for typical Dean numbers with $Pr = 0{\cdot}7$.

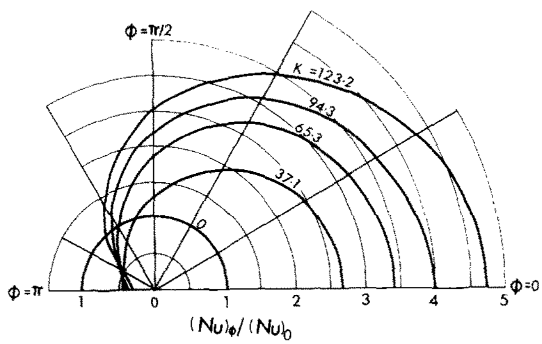

FIG. 2. Local angular distribution of $(Nu)_\phi/(Nu)_0$ with Dean number K as a parameter for $Pr = 0{\cdot}7$.

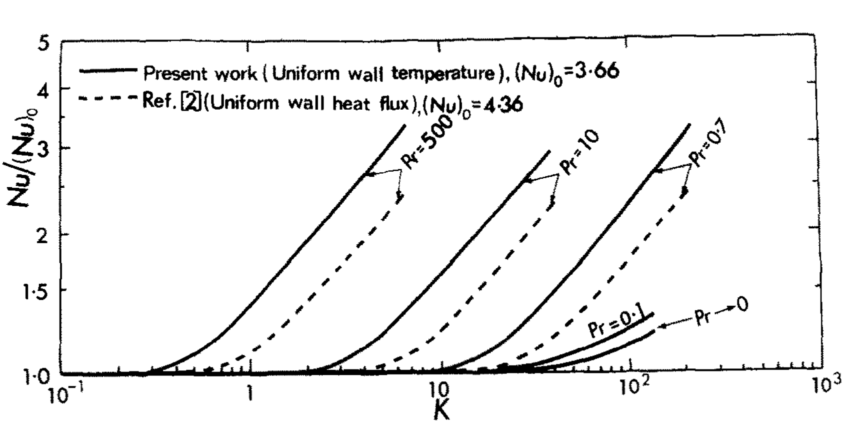

FIG. 3. A comparison between heat transfer results for uniform wall temperature case and uniform wall heat flux case [2] for several Prandtl numbers.

several Prandtl numbers with the results from uniform wall heat flux case [2] included for comparison. The trends of heat transfer results for both cases are seen to be quite similar except for the fact that with a given value of Dean number the value of $Nu/(Nu)_0$ for uniform wall temperature case is always higher than that for uniform wall heat flux case.

3.3 *Comparison with results from approximate analytical methods*

It is desirable to compare the present numerical results with the available results from perturbation method [3, 6] and boundary-layer approximation method [4]. The comparisons are shown in Figs. 4 and 5 for the two thermal boundary conditions in order to assess the adequacy and relative merit of the solution methods used. Maekawa [3] presented results up to $Nu/(Nu)_0 = 1.05$ and further results shown in Fig. 4 are based on computation from the equations for $Nu/(Nu)_0$ given in [3]. It is noted that Maekawa's results do show that at a given value of Dean number, the value of $Nu/(Nu)_0$ for uniform wall temperature case is higher than the corresponding value for uniform wall heat flux case, and this trend is consistent with the numerical results shown in Fig. 4.

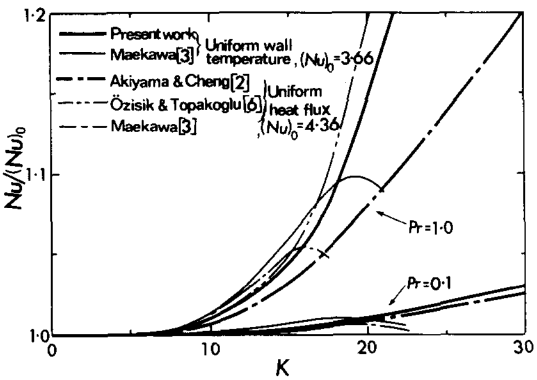

FIG. 4. Comparison between heat transfer results from numerical analysis and perturbation analysis.

Mori and Nakayama [4] conclude that the Nusselt number formula for uniform wall temperature case is the same as that for uniform wall heat flux case. An asymptotic Nusselt number of 16 obtained by Dravid *et al.* [5] for the case of $Pr = 5$, $K = 225$ and $a/R_c = 0.05$ is also shown in Fig. 5 for comparison. It is evident that the correlation equation given by Mori and Nakayama [4] is valid only near $Pr = 1.0$. This observation is similar to that reported in [2] for uniform wall heat flux case. For given Prandtl and Dean numbers, the difference in Nusselt numbers between the uniform wall temperature and uniform wall heat flux is so small that the experimental confirmation would be extremely difficult. Consequently, Mori and Nakayama's conclusion [4] that Nusselt number in curved pipes is hardly affected by the wall temperature condition at high Dean numbers, which is similar to that of turbulent flow in a straight pipe, is only qualitatively correct from a practical viewpoint.

3.4 *A correlation equation for Prandtl number effect*

The possibility for correlating Prandtl number effect on

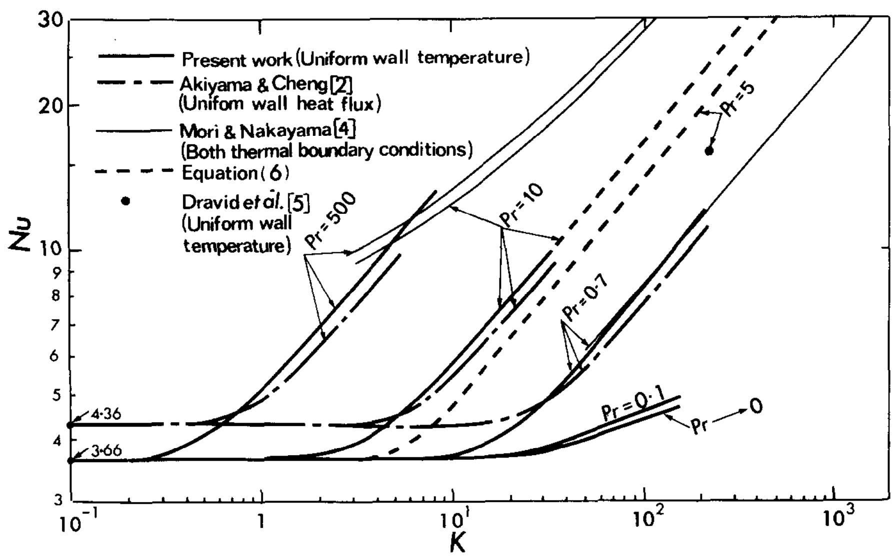

FIG. 5. Comparison of heat transfer results from this work and reference [2] with those from boundary-layer approximation [4].

heat transfer result for fully developed laminar forced convection in curved pipes or channels using a parameter $K^2 Pr$ was pointed out recently [2, 7]. The heat transfer results from this study using the parameter $K\, Pr^{\frac{1}{2}}$ instead of K are shown in Fig. 6, together with Mori and Nakayama's

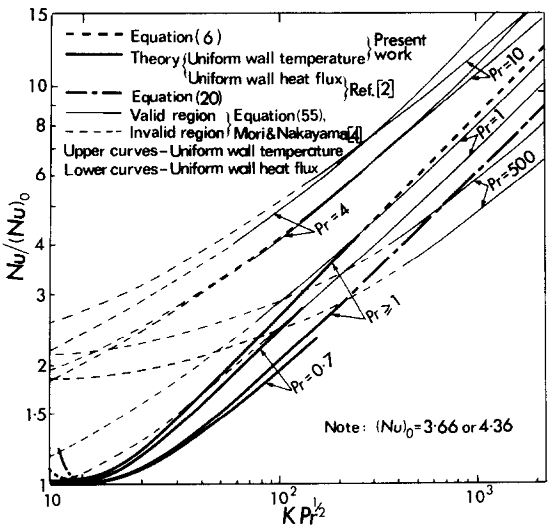

FIG. 6. Proposed correlation curves for Prandtl number effects on heat transfer results valid for $Pr \geqslant 1$ using a parameter $K\, Pr^{\frac{1}{2}}$.

results [4] and the results for uniform wall heat flux case [2] included for comparison. The new correlation is seen to be very effective since all the numerical results for $Pr \geqslant 1$ nearly coincide. The correlation equation for Nusselt number similar to the one reported in [2] is

$$Nu/(Nu)_0 = 0{\cdot}270\, Q(1 - 1{\cdot}48\, Q^{-1} + 23{\cdot}2\, Q^{-2} - 120\, Q^{-3} + 212\, Q^{-4}) \quad (6)$$

where

$$Q = (k\, Pr^{\frac{1}{2}})^{\frac{1}{2}} \geqslant 3{\cdot}0 \quad \text{for } Pr \geqslant 1.$$

As indicated in [2], the regime with $Q \leqslant 3{\cdot}0$ is of no practical importance because of rather weak secondary flow, and the correlation equation (6) can now be regarded as valid for all the practically important laminar flow regimes with sufficient accuracy. In interpreting the results presented in Fig. 6, one should note that $(Nu)_0 = 3{\cdot}66$ for uniform wall temperature and $(Nu)_0 = 4{\cdot}36$ for uniform wall heat flux. Figure 6 also clearly indicates that Mori and Nakayama's Prandtl number effect is incorrect except $Pr \approx 1$.

5. CONCLUDING REMARKS

It has been shown that the heat transfer results for fully developed laminar flow in curved pipes with the two basic thermal boundary conditions of uniform wall temperature and uniform wall heat flux are quite similar, but distinct. For given Prandtl and Dean numbers, the limiting Nusselt number for the uniform wall temperature case becomes higher than that for the uniform wall heat flux case after reaching a certain Dean number. This situation is opposite to that of a pure forced convection in a straight tube. This finding is consistent with the results from perturbation method [3] for extremely low parameter regime; but contradicts the results from boundary-layer approximation [4] which suggests that at high Dean numbers the same formula for Nusselt number can be applied for both thermal conditions.

In applying the present numerical results for heat transfer in curved pipes to design problems, the information on hydrodynamic and thermal entrance lengths [4, 5, 8], transition from laminar to turbulent flow [8], and variable property effect [4] is of practical interest. The present numerical solution is based on constant physical property assumption. When there is a large temperature difference between the inlet and outlet mixed mean temperatures, then the present numerical results may be applied locally for a segment of curved pipe over which the property values may be evaluated using the arithmetic mean bulk temperature between the inlet and outlet of the segment [4]. In this connection, it appears to be more reasonable to define mean heat transfer coefficient based on axial distance instead of temperature difference between the inlet and outlet sections. In addition, the free-convection effect should also be ascertained.

As noted in [2], the assumption of small a/R_c used in the formulation of the problem can be relaxed in practical applications. Recent experiments [9] on laminar flow in curved channels of square section suggest that the present formulation is valid up to $a/R_c = 1/3{\cdot}5$ in practice.

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DROPWISE CONDENSATION OF MERCURY

J. W. ROSE

Department of Mechanical Engineering, Queen Mary College, University of London, London, England

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NOMENCLATURE

$A(r)\mathrm{d}r$, fractional area covered by drops having base radius in the range $r, r + \mathrm{d}r$;
f, condensation coefficient;
g, gravitational acceleration;
h_{fg}, specific enthalpy of phase change;
K_1, a shape factor, see equations (20) and (21);
K_2^*, ratio of base area to curved surface area of a drop;
K_2, defined in equation (22);
K_3, a constant defined in equation (10);
L_p, thickness of promoter layer;
L_0, $2\sigma v_f/h_{fg}$
L_0^*, $L_0 \sin\beta$;
L_3, $\sqrt{[\sigma/g(\rho_f - \rho_g)]}$;
$\dot{m}''$, net mass flux across vapour–liquid interface;
n, a constant in the drop-size distribution, see equation (15);
$\dot{Q}''$, average heat flux for condensing surface;
$\dot{Q}_b''$, heat flux through base area of a drop;
$\dot{Q}_1''$, $\lambda_f T h_{fg}/2\sigma v_f$;
$\dot{Q}_1''^*$, $\dot{Q}_1''/\sin\beta$;
$\dot{Q}_2''$, $h_{fg}^2/\phi v_g\sqrt{(RT)}$;
q, non-dimensionalized heat flux defined in equation (17);
R, specific ideal-gas constant of vapour;
r, base radius of a drop;
$\check{r}$, minimum base radius of a drop;
$\hat{r}$, maximum base radius of a drop;
r_c, radius of curvature of curved surface of a drop;
T, thermodynamic temperature;
v_f, specific volume of condensate;
v_g, specific volume of vapour;
α, vapour-to-surface heat-transfer coefficient;
β, contact angle;
γ, ratio of isobaric to isochoric specific heat capacity;
ΔP, interface pressure difference due to interphase matter transfer;
ΔT, effective vapour-to-surface temperature difference;
ΔT_c, $2\sigma v_f T/r_c h_{fg}$;
ΔT_d, effective temperature difference across a drop due to conduction in the drop, see equation (20);
θ, non-dimensionalized temperature difference defined in equation (18);
θ_0, defined in equation (19);
λ_f, thermal conductivity of condensate;
λ_p, thermal conductivity of promoter layer;
ρ_f, density of condensate;
σ, liquid–vapour interfacial tension;
ϕ, $\Delta P/\dot{m}''\sqrt{(RT)}$.

1. INTRODUCTION

A recent theory of dropwise condensation heat transfer [1] has received strong support from a variety of experimental investigations [2–10] for dropwise condensation of